বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. General Examinations 2022

(Under CBCS Pattern)

Semester - IV

Subject : MATHEMATICS

Paper : SEC 2 - T

Full Marks : 40

Time : 2 Hours

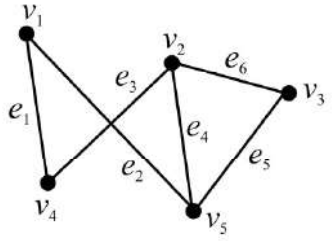
Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

[GRAPH THEORY]

Group - A

Answer any ***four*** questions : 5×4=20

1. Define adjacency matrix of an undirected graph with n vertices. Find the adjacency matrix of the following graph. 2+3



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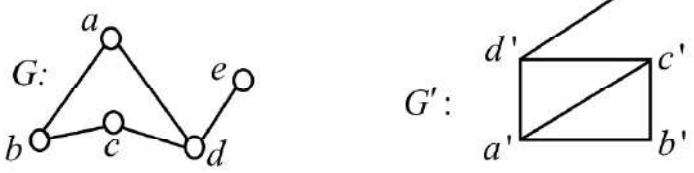
2. Define an Eulerian graph. Prove that a connected graph G is an Eulerian graph if it can be decomposed into circuits. 1+4

3. When two graphs are called isomorphic to each other? Check whether the following two graphs are isomorphic or not. 2+3



4. Define a Hamiltonian graph with an example. Draw a graph which is Eulerian but not Hamiltonian. What are the main differences between Eulerian circuit and Hamiltonian cycle? 2+1+2

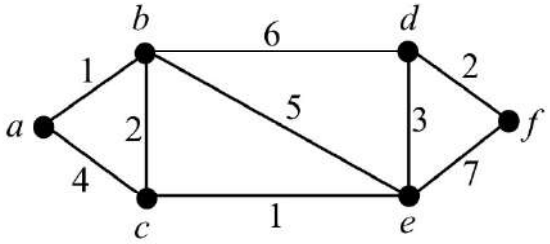
5. Define a pseudo graphs and draw it. Prove that the number of vertices of odd degree in a graph is always even. 1+1+3

6. Define a connected graph. Prove that a graph G is disconnected if and only if its vertex set V can be partitioned into two non-empty, disjoint subsets V_1, V_2 such that there exists no edge in G whose one end vertex is in V_1 and other end vertex is in V_2. 1+4

Group - B

Answer any ***two*** questions : 10×2=20

7. Define the length of a path in a weighted graph. Using Dijkstra's algorithm, find the shortest path between the vertices a and f on the following graph. 2+8



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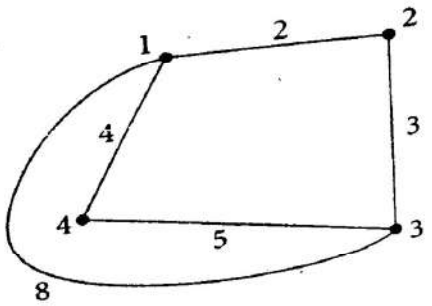
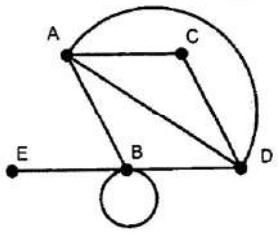
8. Using Floyd–Warshal'sl algorithm find all pair of shortest paths from the following weighted graph :



9. Define graph and write the names of three important graphs. Prove that the degree of a graph is always even. Find the degree of all vertices of the following graph and verify the Euler formula $v - e + f = 2$ for the same graph, where v the number of vertices, e is the number of edges and f is the number of faces (regions bounded by edges, including the outer, infinitely large region). 3+2+2+3



10. Let A, B, C, D, E be the five villages. We would like to connect these villages by a network of pipelines to supply water. The following table shows that the distances in units of 5 km between these five villages. Find a minimal Spanning tree connecting the five villages using Kruskal's algorithm.

	A	B	C	D	E
A	-	2	4	3	5
B	2	-	7	4	6
C	4	7	-	10	8
D	3	4	10	-	9
E	5	6	8	9	-

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OR

[INTEGRAL CALCULAS]

1. Answer any ***four*** questions : 5×4=20

(a) Evaluate : $\int \frac{x^2 dx}{(x-1)^3 (x+1)}$

(b) If $I_{m,n} = \int_0^1 x^m (1-x)^n dx$, where m, n are positive integers, prove that $(m+n+1) I_{m,n} = n I_{m,n-1}$ and hence deduce the value of $I_{m,n}$.

(c) Show that the integral $\iint e^{\frac{y-x}{y+x}} dx\, dy$ taken over the region enclosed by the triangle with vertices at (0, 0), (0, 1), (1, 0) is $\frac{1}{4}\left(e - \frac{1}{e}\right)$.

(d) Show that the length of the arc of the parabola $\frac{l}{r} = 1 + \cos\theta$ cut off by its latus ractum is $l\left\{\sqrt{2} + \log\left(1+\sqrt{2}\right)\right\}$.

(e) Find the area of the segment of the parabola $y = x^2 - 7x + 9$ cut off by the line $y = 3 - 2x$.

(f) Find the volume generated by revolving x-axis, where the area is bounded by $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

2. Answer any ***two*** questions : 10×2=20

(a) Prove that the area included between the folium of Descartes $x^3 + y^3 = 3axy$ and its asymptote $x + y + a = 0$ is equal to the area of its loop.

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(b) (i) Obtain the intrinsic equation of the catenary $y = a\cosh\left(\frac{x}{a}\right)$, taking the vertex $(0, a)$ as the fixed point. 5

(ii) Prove that $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{n^2}{\left(n^2+r^2\right)^{\frac{3}{2}}} = \frac{1}{\sqrt{2}}$. 5

(c) If $I_n = \int \frac{\sin(2n-1)x}{\sin x}\,dx$ and $J_n = \int \frac{\sin^2 n\,x}{\sin^2 x}\,dx$, show that

(i) $n\left(I_{n+1} - I_n\right) = \sin 2nx$; (ii) $J_{n+1} - J_n = I_{n+1}$ 5+5

(d) (i) Prove that $\iiint \frac{dxdydz}{x^2+y^2+(z-2)^2} = \pi\left(2 - \frac{3}{2}\log 3\right)$ extended over the sphere $x^2 + y^2 + z^2 \le 1$. 5

(ii) Find the perimeter of the cardioid $r = a(1-\cos\theta)$. 5

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OR

[MATHEMATICAL FINANCE]

1. Answer any ***four*** questions : 5×4=20

   (a) What are the basic differences between simple and compound interest. Discuss the process of continuous compounding.

   (b) What is present and future value of a stream. Determine the relation between them.

   (c) A young couple has made a non-refundable deposite of first month's rent (equal to $ 1000) on a 6-month apartment lease. The next day they find a different apartment that they like just as well, but its monthly rent is only $ 900. They plan to be in the apartment only 6 months. Should they switch to the new apartment? What if they plan to stay 1 year? Assume an interest rate of 12%.

   (d) Discuss three components of an investor's required rate of return on an investment.

   (e) Briefly discuss about portfolio return. Find the mean and variance of portfolio return.

   (f) Discuss the relationship between Risk and Return.

2. Answer any ***two*** questions : 10×2=20

   (a) Write short note on (i) Business Risk, (ii) Financial risk, (iii) Liquidity risk, (iv) Exchange rate risk (v) Country risk.

   (b) Prove that the expected return μ_i on any asset i satisfies $\mu_i = r_f + \beta_i\left(\mu_M - r_f\right)$, where $\beta_i = \frac{\sigma_{iM}}{\sigma_{M^2}}$ and σ_{iM} is the covariance of the return on asset i and the market portfolio r_m ; $\sigma_{M^2} = Var\left(r_M\right)$.

   (c) Consider two projects whose cash flows are shown in the following table. Find IRR of the two projects and the NPVs at 5%. Show that the IRR and NPV figures yield different recommendation. Can you explain this ?

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	Year					
	0	1	2	3	4	5
Project - 1	–100	30	30	30	30	30
Project - 2	–150	42	42	42	42	42

(d) (i) Your rate of return expectations for the stock of a company during the next year are : 6

Possible rate of return	Probability
–0.6	0.15
–0.3	0.1
–0.1	0.05
0.2	0.4
0.4	0.2
0.8	0.1

Compute expected return on this stock, the variance of this return and its SD.

(ii) An 8% bond with 18 years of maturity has yield of 9%. What is the price of this bond? 4

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